Continuum Electrostatic Probes in the Presence of Negative Ions: A Numerical Solution

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Theme

THE response of a flush-mounted electrostatic probe is studied for a weakly ionized flowing plasma in the presence of negative ions. By working with the governing equations for the entire region rather than the resulting approximating equations in subregions as is commonly done, the charged particle distributions and the electric potential are evaluated directly, without making any prior assumptions about how the solution behaves in separate ambipolar and sheath regions. Numerically calculated current-voltage (CV) characteristics are given for unequal electron and ion temperatures and also curves which exhibit the detailed structure of the sheath region.

Contents

Certain ablating surfaces introduce negative ions into the weakly ionized boundary layers and significantly change the CV characteristics of a flush-mounted electrostatic probe.[1] As is customary, the flow over the probe is divided into subregions and the governing equations in each region are solved separately and the over-all problem is obtained by a suitable matching between regions. When two or more parameters are present in the problem, the asymptotic matching between the sheath and ambipolar regions (with the existence of an essential singularity) becomes impractical. This is the case with electronegative plasmas with unequal electron and ion temperatures.[1] To avoid these difficulties, we solve numerically the governing equation for the entire flow region.[2] For an incompressible, flat-plate flow these constitute the three species conservation equations and the Poisson equation. After employing a boundary-layer approximation, the equations become

$$\tfrac{1}{2} f \beta_s N_s' + [N_s' + \tau_s N_s \psi']' = 0 \tag{1}$$

$$\alpha^2 R \psi'' = \sum_s Z_s N_s \tag{2}$$

where $s = +, -, e$; $N_{+,-,e}$ are the nondimensional number densities of positive ions, negative ions, and electrons, respectively; ψ is the electric potential $(-e\phi/kT_e)$; $R = LU_0/D_+$; α is the Debye ratio (λ_D/L); $\tau = T_e/T_{+,-}$; $\tau_{+,-e}$ are written for convenience such that $\tau_+ = -\tau$, $\tau_- = \tau$, $\tau_e = 1$; β_s represents diffusion coefficient ratios such that $\beta_+ = 1$, $\beta_- = D_+/D_-$ and $\beta_e = D_+/D_e$. Primes denote differentiation with respect to the coordinate $\eta = y(U_0/D_+ x)^{1/2}$ and f is obtained from the Blasius equation $2f''' + ff'' = 0$, which for weakly ionized plasmas is decoupled from the species conservation equations. Equations (1) and (2) are subject to boundary conditions

$$\begin{aligned} &\eta = 0; \quad N_s = 0, \quad \psi = \psi_p \\ &\eta \to \infty; \quad N_+ = 1, \quad N_- = N_{-0}, \quad N_e = 1 - N_{-0}, \\ &\qquad\qquad \psi' = \psi_0' \text{ (outer solution)} \end{aligned} \tag{3}$$

Equations (1) and (2) were approximated by a sequence of linear differential equations by letting $\bar{N}_s$, $\bar{\psi}$ denote approximations to the solution N_s, ψ such that $N_s = \bar{N}_s + n_s$, $\psi = \bar{\psi} + v$. All quadratic terms in n_s and v were neglected and the repeated use

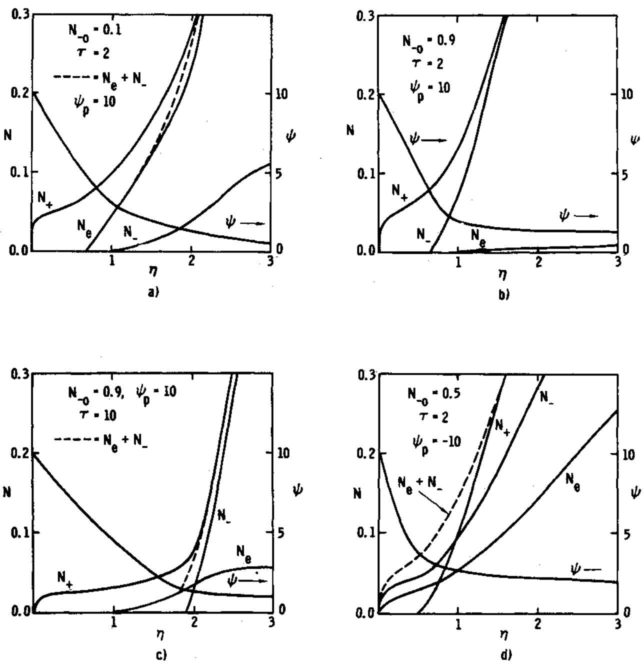

Fig. 1 Sheath structure for various combinations of negative-ion concentrations N_{-0}, electron-to-ion temperature ratio τ and probe potential ψ_p (in all the above, $\alpha^2 R = 0.01$, $\beta_- = 1.0$).

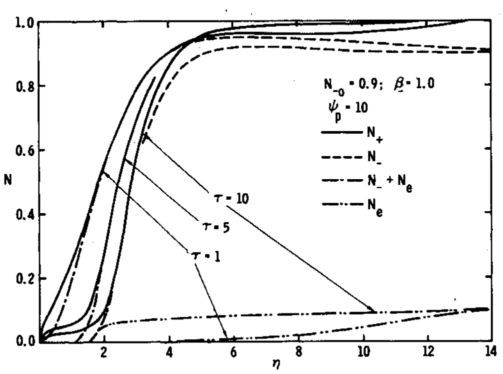

Fig. 2 Charged particle concentration profiles for sheath and ambipolar regions with τ as parameter ($\alpha^2 R = 0.01$).

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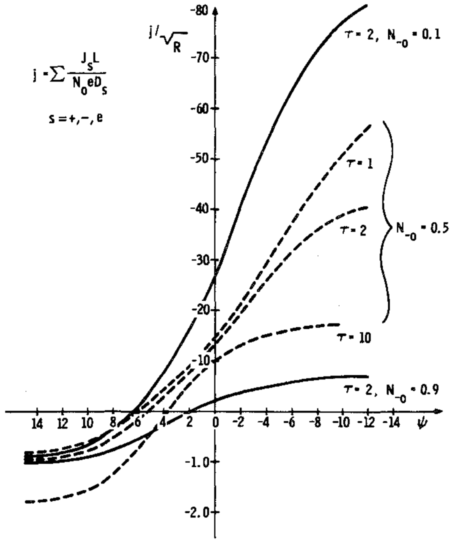

Fig. 3 Calculated current-voltage characteristics for various values of N_{-0} and τ ($\alpha^2 R = 0.01$, $\beta_- = 1.0$).

of the approximation produced a sequence which converged to the solution of N_s, ψ given by the original equations. The linear equations were solved using an implicit finite-difference scheme with variable mesh size to obtain the detailed structure of the thin sheath. Only 6–10 iterations of the linear equations were needed to each step in order to arrive at quite good solutions to the nonlinear equations (1) and (2). Typical run times were 1–2 min on the CDC 6600 computer.

Figures 1–3 summarize the results obtained. For example, Figs. 1a–1d exhibit the details of the sheath structure for two values of the temperature ratio τ and three values of N_{-0}. As τ increases, the sheath size increases slightly ($\lambda_s \propto \tau^{1/6}$), but it "flattens out" considerably, increasing the ion concentration gradients at the sheath edge (Fig. 2) and thus increasing the ion saturation current (Fig. 3). For a given τ and at electron-retarding potentials, the sheath size decreases as the boundary-layer edge negative-ion concentration N_{-0} increases. The CV characteristics calculated for three different negative-ion concentrations (N_{-0}) and three temperature ratios, τ, are shown in Fig. 3. The current densities are defined as follows:

$$j_{\pm,e}/(R)^{1/2} = J_{\pm,e}/(R)^{1/2} N_{-0} e D_{\pm,e} \tag{4}$$

It is evident from the CV characteristics of Fig. 3 that the presence of negative ions suppresses the electron-saturation current and increases slightly the ion-saturation current.

The above calculations show the same qualitative behavior exhibited by the particle concentration curves obtained from the simpler ambipolar solution[1] for $\tau = 1$. For $\tau > 1$, the sheath structure (not included in Ref. 1 calculations) plays a more significant role in determining the saturation currents.

References

[1] Touryan, K. J. and Chung, P. M., "Flush-Mounted Electrostatic Probe in the Presence of Negative Ions," *AIAA Journal*, Vol. 9, No. 3, March 1971, pp. 365–370.

[2] Baum, E. and Chapkis, R. L., "Theory of a Spherical Electrostatic Probe in a Continuum Gas: An Exact Solution," *AIAA Journal*, Vol. 8, No. 6, June 1970, pp. 1073–1077.